

*Semi-Groups and a Class of Singular Perturbation Problems**

ANDREW Y. SCHOENE

Communicated by R. PHILLIPS

Introduction. We assume that A_1 and A_2 are infinitesimal generators of strongly continuous (class (C_0)) semi-groups of bounded linear operators on a Banach space B , and P is a 2×2 matrix with real or complex entries. It follows that $G = \text{diagonal } (A_i) + P$ is the infinitesimal generator of a class (C_0) semi-group on $B \times B$ for which we obtain an explicit representation as a Bochner integral with a Bessel function kernel.

In Section 2 we specialize to

$$G^\epsilon = \frac{1}{\epsilon^{1/2}} \begin{bmatrix} A & 0 \\ 0 & -A \end{bmatrix} + \frac{1}{\epsilon} \begin{bmatrix} -a & a \\ a & -a \end{bmatrix}$$

with A the infinitesimal generator of a group of class (C_0) , and $a > 0$. Each component of the solution of the abstract Cauchy problem $d\bar{u}/dt = G^\epsilon \bar{u}$ satisfies the abstract telegraph equation $\epsilon u_{tt} + 2au_t = A^2 u$: we use semi-group theory to show that this solution converges as $\epsilon \rightarrow 0$ to the solution of $2au_t = A^2 u$ and to obtain a rate of convergence. This is done as follows: the resolvent of G^ϵ , $(\lambda I - G^\epsilon)^{-1}$ is shown to converge at a certain rate to the resolvent of

$$G = \frac{1}{4a} \begin{bmatrix} A^2 & A^2 \\ A^2 & A^2 \end{bmatrix}.$$

The rate of convergence of the resolvents is translated into a rate of convergence of the semi-groups $\exp(G^\epsilon t)$ to the semi-group $\exp(Gt)$ (Theorem 1) via a modified version of the Trotter-Kato Theorem. This in turn implies that the solutions converge (Theorem 2).

In Section 3 we assume A is a self-adjoint operator on a Hilbert space H ; iG^ϵ is then the generator of a unitary group of class (C_0) . The corresponding second order equation, $\epsilon u_{tt} + 2aiu_t + A^2 u$ is a transformed version of the Klein-Gordon

* This research was wholly supported by NSF Grant GP-8856 and based on work submitted in partial fulfillment of the requirements for the PH.D. degree at the University of New Mexico.