

Existence of Solutions of the Equilibrium Equations for Nonlinearly Elastic Rings and Arches

STUART ANTMAN

Communicated by IAN SNEDDON

Introduction. In this paper we study the nonlinear sixth order system of ordinary differential equations for the equilibrium of extensible nonlinearly elastic rings and arches under hydrostatic pressure. The material of these rods is characterized by a strain energy function that depends upon a measure of stretching and a measure of bending. By the direct methods of the calculus of variations, we prove the existence and regularity of at least one solution of these equations for each value of the change in area “enclosed” by the ring or arch. In a like manner, we furnish a sufficient condition on the response of the material to tension that ensures the existence of at least one classical solution for each value of the pressure. We also show that the initially circular uniform ring has at least one noncircular solution for certain ranges of pressure.

This work represents a marked improvement over that of [2] because it treats in detail three questions not discussed there, namely, the satisfaction of the continuity condition, the regularity of solutions, and the existence of buckled states of circular rings. (The continuity condition, expressing the impossibility of compressing a segment of the rod of nonzero length into one of zero length, appears as a strict inequality for one of the dependent variables of the problem. The presence of a strict inequality among the conditions defining the admissibility class of functions for a variational problem makes it questionable whether the function that renders the variational functional an infimum actually belongs to the admissibility class. In an abstract sense, the functions that satisfy such a unilateral constraint do not form a subset of a linear space that is convenient for the purposes of variational methods.) In this paper we show that the introduction of a mild and physically reasonable hypothesis on the material response enables us to reformulate our problem so that both the continuity condition and the regularity theory can be handled in a natural way. Moreover we employ a somewhat more convenient function space than that of [2] and we refine and simplify some proofs that have analogs in [2].