

# *A Nonlinear Version of the Principle of Limiting Amplitude*

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**1. Introduction.** If a dynamical system is governed by a linear equation and driven by an externally imposed harmonic force and if there is some kind of dissipation, then for large times the system is expected to tend to an harmonic motion, with the period of the external force, regardless of its initial configuration. This is called the Principle of Limiting Amplitude. A classical example of this behaviour is furnished by the motion of a damped spring subject to an harmonically varying external force.

Suppose now, that the equation of motion in the above system is replaced by a nonlinear equation. Unlike linear differential equations, nonlinear equations often do not admit solutions which can be continuously extended whenever the equations themselves remain regular. In addition, although the principle of superposition holds for linear waves, it is not valid for nonlinear waves. Thus, in general, one can only expect this system to tend in some generalized sense to a periodic motion of the same period as the external force and then only when an additional condition; such as that the amplitude of the forcing term be sufficiently small, is imposed on it.

It is our purpose here to prove such a result and to obtain an estimate on the rate at which the periodic motion is approached for a system governed by the nonlinear equation of evolution

$$(1-1) \quad u_{,t}(t) + A(t)u(t) + du_{,t}(t) = f(t),$$

in the interior of a finite reflecting body  $\Omega$  where the forcing term  $f(t)$  is periodic in  $t$  of period  $\tau$ ,  $d$  is a real constant called the damping coefficient,  $u(t)$  is a vector-valued function defined on a subinterval of the real line into a space of complex-valued functions  $u(x)$  defined on a bounded domain  $\Omega$  in the real Euclidean

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