

Combinatorics of Partial Sums with Applications to Limit Theorems

HERMANN DINGES

Communicated by G.-C. ROTA

Some arguments in the proof of the ergodic limit theorems have always been understood to be of a combinatorial nature. Besides the celebrated maximal ergodic theorem a certain smoothing procedure due to Chacon and Ornstein is an extremely important device of this type.

Something like these theorems is presented here in purely combinatorial terms. The applications given in the second part of the paper should give an idea how the results can be used in measure theory. General ergodic limit theorems will be derived in a forthcoming paper using this combinatorics together with a theory of decomposition of joint distributions, developed by H. Rost and the author.

I. Combinatorics. Let Ω be a space and let $s_{i+1}^i, s_{i+2}^{i+1}, \dots$ be a sequence of real valued functions on Ω .

Denote (with max and min defined pointwise)

- 1) $s_k^i = s_{i+1}^i + s_{i+2}^{i+1} + \dots + s_k^{k-1}$ for $i < k$, $s_i^i = 0$ for all i .
- 2) $*h_k^i = \max \{s_{i+1}^i, s_{i+2}^{i+1}, \dots, s_k^i\}$ for $i < k$.
- 3) $*l_k^i = -\min \{s_k^{k-1}, s_k^{k-2}, \dots, s_k^i\}$ for $i < k$.
- 4) $h_k^i = (*h_k^i)^+ = \max \{s_i^i, s_{i+1}^i, \dots, s_k^i\}$, $h_i^i = 0$.
- 5) $l_k^i = (*l_k^i)^+ = \max \{-s_k^k, -s_k^{k-1}, \dots, -s_k^i\}$, $l_k^k = 0$.
- 6) $d_k^i = (*h_k^i)^-$ for $i < k$.
- 7) $g_k^i = (*l_k^i)^-$ for $i < k$.
- 8) $\tilde{l}_k^i = \max \{s_k^k, s_k^{k-1}, \dots, s_k^i\}$ for $i \leq k$.
- 9) $\tilde{g}_k^i = \min \{s_i^i, s_{i+1}^i, \dots, s_{i-1}^i\} - \min \{s_i^i, \dots, s_k^i\}$ for $i < k$.

Remark 1. The functions $h_k^i, l_k^i, g_k^i, d_k^i$ are nonnegative. \tilde{l}_k^i and \tilde{g}_k^i are obtained from the sequence $\{s_{i+1}^i = -s_{i+1}^i\}$ in the same way as l_k^i and g_k^i are obtained from $\{s_{i+1}^i\}$.