

On the Movement of a Chemical Reaction Interface

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Introduction. This paper is concerned with a free boundary problem in several space variables for parabolic partial differential equations having the form

$$\alpha(u) \frac{\partial u}{\partial t} = \operatorname{div} k(u) \operatorname{grad} u,$$

A precise formulation is given in Section 1. The solution u is required to vanish at the free boundary Γ and, moreover, a condition of the type

$$\nu k_1(u_1) \frac{\partial u_1}{\partial n} + k_2(u_2) \frac{\partial u_2}{\partial n} = 0 \quad \text{on } \Gamma$$

is imposed, where the subscripts 1 and 2 refer to the two sides of Γ .

In the one dimensional case the problem treated here was formulated in the literature as early as 1928 in several different contexts: In the theory of diffusion flames [1] and in the study of the diffusion of oxygen and lactic acid through tissue [10]. In the case of diffusion flames, the free boundary Γ represents the position of a flame created by the combustion of gaseous fuel with oxygen along an interface where the two substances come together. In the case of [10], Γ represents the interface between lactic acid and oxygen along which a chemical reaction occurs. The problem treated here can also be thought of as a mathematical model for the following situation: Two substances in solution diffuse in, say, a beaker. When they come together at Γ they chemically react quickly and completely (and perhaps precipitate out of solution). Hence at Γ the concentrations are zero. A certain amount of substance 1 must flow in from one side of Γ in order to react with a given amount of substance 2 flowing in from the opposite side of Γ .

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