

# *A Removable Singularity Theorem for Local Homeomorphisms*

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**1. Introduction.** Let  $M$  and  $N$  be manifolds and  $f: M \rightarrow N$  a continuous mapping. The set  $B_f$  of points  $p \in M$  at which  $f$  is not a local homeomorphism is called the *branch set*. Church and Hemmingsen, [1], have shown that if  $m = \dim M \geq 3$ , then  $B_f$  is a perfect set. This result is false for  $m = 2$  because  $M \sim \{p\}$  is in general not simply connected.

Interpreted in a different way, the Church–Hemmingsen result becomes a local removable singularity theorem: a possible branch point which is isolated is not a branch point at all. From this point of view it is natural (and in fact desirable for the theory of quasiconformal functions in space) to ask for a removable singularity theorem of this type where the function is not assumed to be continuous or even defined at the isolated point in question.

It is our intention here to provide such a theorem in the case  $N = \mathbf{R}^n$ : to present a necessary and sufficient condition that a local homeomorphism  $f$  in  $M \sim \{p\}$  can be so defined at  $p$  to become a local homeomorphism of  $M$ . Our proof will be independent of the work of Church and Hemmingsen in that we do not first prove that  $f$  is continuous at  $p$ . Rather, we obtain the full strength of the conclusion at one stroke.

**2. Notation and statement of results.** For our purposes it suffices to work in  $n$ -dimensional Euclidean space  $\mathcal{R}^n$ ,  $n \geq 3$ , and the corresponding Möbius space  $\bar{\mathcal{R}}^n = \mathcal{R}^n \cup \{\infty\}$ .  $\mathcal{B}$  will denote the punctured open unit ball  $\{x: 0 < |x| < 1\}$  and  $\mathcal{S}$  the  $(n - 1)$ -sphere  $\{x: |x| = 1\}$ .

The modulus  $M(\Gamma)$  of a family  $\Gamma$  of curves in  $\bar{\mathcal{R}}^n$  is defined as follows. We say that a non-negative Borel measurable function  $\rho$  is admissible for  $\Gamma$ , and write  $\rho \in \mathcal{G}(\Gamma)$  if

$$\int_{\gamma} \rho \, ds \geq 1,$$

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