

Structural Stability and the Störmer Problem

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Introduction. The motion of a charged particle under the influence of the earth's magnetic field may be calculated to first approximation by considering the field to be equivalent to that of a magnetic dipole situated at the center of the earth. The question of existence and uniqueness (for fixed values of energy and angular momentum) of an orbit entering the dipole has long been of interest to mathematicians and physicists. In the early 1900's, Störmer [4] found a formal series expansion, which, if it converged, would represent a trajectory running into the singularity. From this expansion, and many numerical calculations, he was led to conjecture the existence and *uniqueness* of a "Störmer orbit" entering the dipole. In 1944 the Swedish mathematician Malmquist [3] succeeded in proving the existence of at least one such solution whose asymptotic expansion was given by the formal series of Störmer. In this paper we will first present an alternate proof for the existence of a Störmer orbit, and then establish its uniqueness. We will then be able to "read off" the behavior of all trajectories in the vicinity of the dipole. Moreover, our method will be applicable to more general Hamiltonian systems of two degrees of freedom.

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1. A fundamental theorem.

a. In this section we prove a theorem concerning the behavior of solutions of a differential equation in the neighborhood of an equilibrium point, and in Section 2 we will apply this theorem to the dipole field. We state our result as follows:

Theorem. *Let q, y_1, y_2 be three real coordinates and set $y = y_1 + iy_2, r =$*

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