

# *The Square-integrability of Operator-valued Functions with Respect to a Non-negative Operator-valued Measure and The Kolmogorov Isomorphism Theorem\**

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**1. Introduction.** Let  $\mathcal{H}$  and  $\mathcal{K}$  be two separable Hilbert spaces, and  $\mathfrak{F}$  be a  $\delta$ -ring of subsets of a space  $\Omega$  (a  $\delta$ -ring is a ring closed under countable intersection). Let  $M$  be a countably additive function defined on  $\mathfrak{F}$  with values in the class  $T(\mathcal{H}, \mathcal{K})$  of all non-negative definite compact operators of finite trace on  $\mathcal{H}$  to  $\mathcal{K}$ . In the case  $\mathcal{H}$  and  $\mathcal{K}$  are the Euclidean spaces  $R^q$  and  $R^p$  ( $1 \leq p, q < \infty$ ) respectively, the integral

$$(1.1) \quad \int_{\Omega} \Phi \, dM \, \Psi^*$$

is defined in such a way that the space  $L_{2,M}$  of functions with values linear operators on  $R^q$  into  $R^p$  and square integrable with respect to  $M$ , *i.e.* the space

$$(1.2) \quad L_{2,M} = \left\{ \Phi : \text{trace} \int_{\Omega} \Phi \, dM \, \Phi^* \text{ finite} \right\}$$

is a complete inner product space ([21]) under the norm

$$(1.3) \quad \|\Phi\|_M = \text{trace} \int_{\Omega} \Phi \, dM \, \Phi^*.$$

For the special case  $p = q = 1$ , the completeness of  $L_{2,M}$  is the core of the celebrated Riesz–Fisher Theorem.

The importance of the completeness of  $L_{2,M}$  along with the denseness of the simple functions in it in connection with multivariate prediction theory of

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