

On the Existence of Equilibrium States of Thin Elastic Shells (I)

MELVYN S. BERGER*

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The equilibrium states of thin shallow elastic shells under the action of applied forces can be determined by solving a system of two quasilinear elliptic partial differential equations, subject to appropriate boundary conditions, (the so-called von Kármán equations). Experience indicates that the nonlinear nature of these equations is necessary for a mathematical understanding of the observed deformations. In this paper we shall make a qualitative study of the structure of the solutions of the above mentioned equations with a view to establishing rigorously certain observed facts concerning the associated bending and buckling problems. In case the shell is initially flat (*i.e.* a plate), the associated equilibrium states have been studied by P. Fife and the author in [1], [2], and [3]. The mathematical analysis in that case is considerably simplified owing to the well known experimental fact that the relevant equilibrium states can be calculated approximately by linearized equations (for small deformations). This linearization is not valid for elastic shells. Nonetheless we shall show that a “stability bound” (*cf.* Theorem 4.1) for equilibrium states can be computed by a somewhat different linearization procedure.

This article is arranged in the following manner: Section 1 states and formulates the boundary value problems to be considered here. Section 2 reformulates the solutions of these problems as critical points of associated variational problems. In Sections 3 and 4 we study questions concerning the existence and multiplicity of the above mentioned critical points for the bending and buckling problems, respectively.

Section 1. Formulation and Statement of Problems. We consider a thin shallow shell S of arbitrary shape whose plane projection is a bounded domain Ω in the xy plane with boundary $\partial\Omega$. Suppose the shell is acted on by an external force $Z(x, y)$ and by forces along $\partial\Omega$. Then, subject to appropriate boundary

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