

The Vorticity at Infinity for Solutions of the Stationary Navier–Stokes Equations in Exterior Domains

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1. Introduction. This paper is concerned with the asymptotic behavior of the vorticity of solutions of the stationary Navier–Stokes equations in a neighborhood of infinity in either three or two dimensions. It is assumed that the velocity field $\mathbf{w}(x)$ tends to a non-zero limit \mathbf{w}_0 at infinity. The Navier–Stokes Equations represent the flow of a viscous incompressible fluid.

In Section 2 we give definitions and notation and present without proof some results necessary for our developments. The results for three dimensions are presented in Subsection 2.1, and those for two dimensions, which are roughly parallel, in Subsection 2.2. In particular we give some integral representations and introduce classes PR_3 and PR_2 of “physically reasonable” solutions, which, respectively, could be characterized as consisting of solutions $\mathbf{w}(x)$ such that $\mathbf{w}(x) - \mathbf{w}_0 = O(|x|^{-1})$ at infinity if $n = 3$, or $\mathbf{w}(x) - \mathbf{w}_0 = O(|x|^{-1/2})$ at infinity if $n = 2$. These classes were investigated by Finn [1, 6], Smith [5], and Finn and Smith [10], who, in particular, showed that they correspond to the expected behavior of a physical flow past an obstacle. They will be the only classes considered in this paper. Under weaker assumptions our methods do not apply.

In Section 3 we consider the vorticity $\omega(x) \equiv \nabla \times \mathbf{w}(x)$ of any class PR solution $\mathbf{w}(x)$. There are physical grounds for expecting that the vorticity decays more rapidly than any negative power of $|x|$ as $x \rightarrow \infty$ along any ray not in the direction of \mathbf{w}_0 , while in the direction of \mathbf{w}_0 there is a paraboloidal (parabolic) “wake region”, where the decay is less rapid. (See Chang [4, p. 833]). Outside of this region the flow is nearly a potential flow. Chang [4] uses an assumption of this type in deriving expansions for the velocity at large distances from the body in two-dimensional problems.

The principal results include the following (see Theorem 3.5): let $\mathbf{u}(x) \in \text{PR}_3$ in a region exterior to a smooth, bounded, closed surface Σ ; let