

# The Growth of the Nevanlinna Proximity Function and the Logarithmic Potential

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**1. Introduction.** Let  $f(z)$  be a nonconstant entire function. We shall study the growth of the Nevanlinna proximity function

$$m(r, a) = m(r, a, f) = \frac{1}{2\pi} \int_0^{2\pi} \log^+ \frac{1}{|f(re^{i\theta}) - a|} d\theta,$$

for varying values of  $a$ . In addition to the standard conventions and notations of Nevanlinna's theory of meromorphic functions, we assume some familiarity with basic definitions of the theory of the (logarithmic) potential (cf. [15; Ch. III]). Indeed, it is a connection between these two studies which forms the basis of this work.

We consider the following problem:

If  $f(z)$  is of order  $\rho (\leq \infty)$ , determine for how large a set  $A$  can

$$(1.1) \quad \lim_{r \rightarrow \infty} m(r, a) = \infty \quad \text{for every } a \in A.$$

Our answer is contained in two theorems. The first considers the case  $\rho > \frac{1}{2}$ :

**Theorem 1.** *Given any set  $A$  of capacity zero and  $\frac{1}{2} < \rho \leq \infty$ , there exists an entire function of order  $\rho$  such that (1.1) is satisfied.*

In §2 we shall see that (1.1) cannot hold for sets  $A$  "larger" than of capacity zero; thus Theorem 1 is best possible. On the other hand, if  $\rho < \frac{1}{2}$ , no function can satisfy (1.1) if  $A$  is nonempty; in fact, the "cos  $\pi\rho$  theorem" [14; p. 275] asserts that a nonconstant entire function of order  $\rho < \frac{1}{2}$  cannot have bounded minimum modulus.

It thus remains to consider what happens if  $\rho = \frac{1}{2}$ . While the set  $A$  determined by (1.1) now need not be empty, we find that it must be a "small" set of capacity zero (see Remark (A) of §9):

**Theorem 2.** *Let  $A$  be a set in the complex plane. Then there exists an entire*

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