

Some Dimension Theorems for the Sample Functions of Stable Processes

JOHN HAWKES

Communicated by R. GETTOOR

1. **Introduction.** Let $X_t(\omega)$ be a stable process of index α , $0 < \alpha \leq 2$, in R^d and let $X(B, \omega)$ denote the image under $X_t(\omega)$ of a time set B . In [1] R. M. Blumenthal and R. K. Gettoor established the following relationship (being a generalization of a result due to H. P. McKean, [7], for Brownian motion) between the dimensions of $X(B, \omega)$ and B

$$P[\dim X(B, \omega) = \min(\alpha \dim B, d)] = 1,$$

for all Borel subsets, B , of $[0, 1]$.

In some circumstances we may have a time set $S(\omega)$, depending on ω , and may wish to know the dimension of $X[S(\omega), \omega]$. In this case the above result does not help. In [2] we showed that if we had the stable subordinator of index α , $T_t(\omega)$, then the sample functions were uniformly increasing and the following result held:

$$P[\dim T(B, \omega) \geq \alpha \dim B \text{ for all Borel sets } B] = 1.$$

We then used this result in [2] and [3] to obtain new dimension results. In Theorem 1, below, we establish the somewhat surprising fact that this result is true for a much wider class of stable processes. This enables us to prove results similar to those obtained in [2] and [3] and, in particular, to obtain a satisfactory solution to the problem of the dimension of the intersection of the range of a stable process and a bounded Borel set.

In Section 2 we introduce a few definitions and quote some Lemmas. In Section 3 we prove Theorem 1 and in Section 4 we deduce some interesting dimension theorems.

2. **Preliminaries.** The d -dimensional characteristic function of $X_t(\omega)$ has the form $\exp[-t\psi(z)]$ where

$$\psi(z) = i(a, z) - \lambda |z|^\alpha \int_{S^d} w_\alpha(z, \theta) m(d\theta),$$