

On Partially Mixing Transformations

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Communicated by G. C. ROTA

1. Introduction. Let (X, \mathcal{G}, m) be the unit interval with Lebesgue measure, and let τ be a measure preserving transformation on X . τ is *mixing* if

$$(1.1) \quad \lim m(\tau^n A \cap B) = m(A)m(B), \quad A \text{ and } B \text{ in } \mathcal{G}.$$

It is clear that (1.1) implies

$$(1.2) \quad \liminf m(\tau^n A \cap B) > 0, \quad m(A)m(B) > 0.$$

Our purpose in this paper is to give an example which shows that (1.2) does not imply (1.1).

Let us say that τ is *partially mixing* if there exists $\alpha > 0$ such that

$$(1.3) \quad \liminf m(\tau^n A \cap B) \geq \alpha m(A)m(B), \quad A \text{ and } B \text{ in } \mathcal{G}.$$

Our main result is that there exists a partially mixing transformation that is not mixing.

Although partial mixing does not imply mixing, a result due to England and Martin [2] shows that partial mixing does imply weak mixing. τ is *weakly mixing* if

$$(1.4) \quad \lim_{n_i} m(\tau^{n_i} A \cap B) = m(A)m(B), \quad A \text{ and } B \text{ in } \mathcal{G},$$

where (n_i) is a sequence of positive integers of density one, which generally depends on A and B . Now (1.4) implies

$$(1.5) \quad \liminf_{n_i} m(\tau^{n_i} A \cap B) > 0, \quad m(A)m(B) > 0,$$

where (n_i) is as above. As a corollary of the result in [2], it follows that (1.5) does imply (1.4). Note that (1.1) and (1.2) are analogous to (1.4) and (1.5). On the other hand, a construction due to Chacon [1] yields an example of a transformation that is weakly mixing but not partially mixing. Thus partial mixing lies between weak mixing and mixing.

Our method for constructing a partially mixing transformation utilizes a construction in [3]. In §2 we briefly summarize the construction in [3] in terms