

On the Asymptotic Behavior of Solutions of the Prandtl Boundary Layer Equations

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1. Introduction. We consider the Prandtl equations for steady laminar two-dimensional flow past a rigid body [6]

$$(1) \quad uu_x + vv_y = \nu u_{yy} + UU'$$

$$(2) \quad u_x + v_y = 0.$$

Here x and y are orthogonal coordinates, with x representing arc length along the body and y the distance from the body; u and v are the velocity components in the x -direction respectively y -direction. The positive constant ν is the kinematic viscosity, $U(x)$ is the given velocity of the outer flow (free stream velocity). Equations (1) and (2) are considered in the region $0 \leq x \leq a, 0 < y < \infty$ ($a > 0$). We have the following boundary conditions

$$(3) \quad u(x, 0) = 0, \quad v(x, 0) = v_0(x) \quad \text{for } 0 \leq x \leq a$$

$$u(0, y) = u_0(y) \quad \text{for } 0 \leq y < \infty,$$

where $v_0(x)$ and $u_0(y)$ are given functions. Furthermore, a boundary condition at infinity

$$(4) \quad u(x, y) \rightarrow U(x) \text{ as } y \rightarrow \infty \text{ uniformly in } x \in [0, a]$$

is imposed.

The main purpose of this paper is to give various estimates on the rate of convergence of $u(x, y) \rightarrow U(x)$ as $y \rightarrow \infty$, depending on the rate of convergence of $u_0(y) = u(0, y) \rightarrow U(0)$. In several cases estimates from above and below are obtained, *i.e.*, the rate of convergence is precisely determined. A first result in the direction of our research is the fact that under rather general assumption on u the boundary condition at infinity (4) is always satisfied. It is stated in Theorem 3, together with some bibliographical remarks.

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