

Distributional Boundary Values of Functions Analytic in Tubular Radial Domains

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I. Introduction. Tillmann [1] has characterized functions $f(z)$ which are analytic in an octant $G_\delta = \{z: \delta_j \operatorname{Im}(z_j) > 0, \delta = (\delta_1, \dots, \delta_n), \delta_j = \pm 1, j = 1, \dots, n\}$ and which have distributional boundary values in S' , the space of tempered distributions. He has shown that such functions must satisfy the boundedness condition

$$(1) \quad |f(z)| \leq M \prod_{j=1}^n (1 + |z_j|^2)^{m_j} |y_j|^{-1/2-k_j},$$

where $z = (z_1, \dots, z_n) = (x_1 + iy_1, \dots, x_n + iy_n)$ and M, m_j, k_j are constants. Working independently Vladimirov [2] characterized distributional boundary values in S' using the class of functions $K^+(K^-)$, which are functions that are analytic in tubular domain $T^+(T^-) = \mathbf{R}^{n+1} + i\Gamma^+(\Gamma^-)$, $\Gamma^+(\Gamma^-)$ being the future (past) light cone, and which satisfy a boundedness condition similar to (1). Vladimirov further showed that the S' boundary value of such a function $f(z)$ must be the Fourier transform of an element $U \in S'$ which vanishes outside $\bar{\Gamma}^+(\bar{\Gamma}^-)$, and $f(z)$ has the representation $f(z) = \langle U, e^{i(z, t)} \rangle$. Later Vladimirov [3] extended his results to the more general situation that $f(z)$ be analytic in a tubular cone of which a tubular domain is an example. In [4] Vladimirov has summarized his results in their most general form.

Several authors have obtained distributional boundary value results in S' using a boundedness condition that is less restrictive than that of Tillmann and Vladimirov. Beltrami and Wohlers [5], [6], [7] have shown that if $f(z)$, $z \in \mathbf{C}^1$, is analytic in $\operatorname{Im}(z) > 0$, satisfies

$$(2) \quad |f(z)| \leq C_\delta (1 + |z|)^p, \quad \operatorname{Im}(z) \geq \delta > 0,$$

and converges in the S' topology to a generalized function U , then $U \in S'$, U is the Fourier transform of an element $V \in S'$ which has support in $[0, \infty)$, and $f(z)$ is the Laplace transform of V . The same result was obtained by DeJager [8] in a slightly more general setting. The extension to n dimensions has been