

An Inversion Formula for Analytic Families of General Wiener-Hopf Operators

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1. Let H be a separable Hilbert space, A a linear operator on H , and P an orthogonal projection onto the subspace $R(P)$ of H . A general Wiener-Hopf operator is any operator of the form

$$T_P(A) = PA \mid R(P),$$

the vertical bar denoting restriction. There are a great many applications in which one wishes to solve an equation

$$T_P(A)x = y;$$

because of this, it is of interest to derive a formula for $T_P^{-1}(A)$, the inverse of $T_P(A)$.

I derived such a formula in [2] and [3], assuming that A is positive and self-adjoint. This formula is

$$(1.1) \quad T_P^{-1}(A)y = \sum (y, \chi_n)\chi_n,$$

where parentheses denote the scalar product in H , and $\{\chi_n\}$ is any sequence in H with the following properties:

$$(1.2) \quad \text{sp } \{\chi_n\} = R(P)$$

and

$$(1.3) \quad \{A^{1/2}\chi_n\} \text{ is orthonormal.}$$

Notice that if it happens that $T_P(A)$ has a sequence of eigenfunctions, complete in $R(P)$, $\{\chi_n\}$ can be taken to be that sequence. In that case, (1.1) is simply the usual eigenfunction expansion for $T_P^{-1}(A)$. On the other hand, (1.1) is valid whether or not $T_P(A)$ possesses such a sequence of eigenfunctions. Notice also that, given any positive, self-adjoint operator A and any projection P , the sequence $\{\chi_n\}$ needed in (1.1) can be computed by selecting any sequence $\{\omega_n\}$ whose span is $R(P)$, and then orthonormalizing $\{A^{1/2}\omega_n\}$ by the Gram-Schmidt process. The fact that the sequence $\{\chi_n\}$ can be computed so simply was used in [4] to solve certain integral equations that arise in applications.

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