

Multiple Solutions for a Class of Nonlinear Boundary Value Problems

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1. Introduction. This paper is concerned with the existence of multiple solutions of variational boundary value problems of the following general form:

$$(1.1) \quad \operatorname{div} (a(x) \operatorname{grad} u) + c(x)u - b(x, u) = 0, \quad u = 0 \text{ on } \partial\Omega.$$

Ω is a bounded region in Euclidean n -space. $a(x)$ is a measurable bounded symmetric $(n \times n)$ matrix with least eigenvalue greater than $\nu > 0$ for all $x \in \Omega$. $c(x)$ is a non-negative function belonging to $L_\rho(\Omega)$ with $\rho > n/2$. $b(x, t)$ is measurable in x , continuous in t , and satisfies the following conditions.

- (b.i) $b(x, -t) = -b(x, t)$ for all t and almost all x .
- (b.ii) $tb(x, t) > 0$ for all $t \neq 0$ and almost all x .
- (b.iii) For almost all x , $t^{-1}b(x, t)$ is an increasing function of $|t|$ such that

$$\lim_{t \rightarrow 0} t^{-1}b(x, t) = 0; \quad \lim_{t \rightarrow \infty} t^{-1}b(x, t) \geq c(x).$$

The second of these limits is allowed to be $+\infty$.

- (b.iv) If $n > 1$, $|b(x, t)| \leq m(x) + k|t|^r$ where $m(x) \in L_q(\Omega)$, $q = 2n/(n+2)$; $r < (n+2)/(n-2)$. (If $n = 2$ and b is bounded in x it suffices to assume that $|b(x, t)|$ grows less rapidly than e^{t^2} .)

In Theorems 2 and 4, but not 1, 3 and 5, we also assume

- (b.v) For almost all x , $b(x, t)$ is of class C^2 in t . Denote the first and second partial derivatives with respect to t by b' and b'' respectively. We assume that $b'(x, t) > t^{-1}b(x, t)$ for $t \neq 0$ and almost all x , and, if $n > 1$,

$$|b'(x, t)| \leq m_1(x) + k_1|t|^{r-1}; \quad |tb''(x, t)| \leq m_2(x) + k_2|t|^{r-1},$$

where $m_1(x), m_2(x) \in L_{n/2}(\Omega)$; $r < (n+2)/(n-2)$.

Closely related to (1.1) is the linear characteristic value problem

$$(1.2) \quad \operatorname{div} (a(x) \operatorname{grad} w) + \mu c(x)w = 0, \quad w = 0 \text{ on } \partial\Omega.$$