

# *Elastic-Plastic Torsion of Simply Connected Cylindrical Bars\**

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**1. Introduction.** This is a continuation of previous works to present comprehensive results for elastic-plastic torsion of a cylindrical bar with simply connected cross sections. In order to include the important case of a circular shaft with key ways the present generalization is indeed necessary. It so happens that the stress function for elastic-plastic torsion depends upon the geometric nature of the cross section  $G$  of the bar in a rather sensitive way. Accordingly, we state our restrictions on  $G$  explicitly as follows: (i)  $G$  is a Jordan domain satisfying the well-known cone condition [23], (ii)  $\partial G$ , boundary of  $G$ , possesses continuous non-zero radius of curvature except at a finite number of corners and (iii) between any two neighbouring corners, the curvature of  $\partial G$  possesses only a finite number of maximum and minimum, [18].

According to von Mises [1], the elastic-plastic torsion problem is to find a function  $\psi(x, y)$  in  $C_0^0(\bar{G}) \cap C^1(G)$  such that (i)  $|\text{grad } \psi| \leq k$  a positive constant  $k$  in  $G$  and (ii) wherever  $|\text{grad } \psi| < k$ ,  $\psi$  must be twice differentiable and satisfies the equation  $\Delta\psi = -2\mu\theta_0$  there. Here  $\theta_0$  and  $\mu$  are positive constants.

**2. Variational formulation.** The problem as was stated above has never been attacked directly. Also its uniqueness of solution remains an open problem. As was done in [2] and [19], we shall formulate it as a variational minimum problem with an inequality side constraint. It will be shown that the extremal not only solves the elastic-plastic torsion problem but also furnishes various other information through its minimizing properties.

According to the results in [18], the solution to the corresponding completely plastic torsion problem is given by

$$(2.1) \quad \Psi(q) = k\rho(q, \partial G), \quad q \text{ in } \bar{G}.$$

Let  $C_0^\infty(G)$  be the class of infinitely differentiable functions with compact support in  $G$ . For every  $f$  in  $C_0^\infty(G)$ , we denote by  $\|f\|_1$  the Dirichlet norm of  $f$ , i.e.,

$$(2.2) \quad \|f\|_1^2 \equiv \iint_G [|\text{grad } f|^2 + f^2] \, dx \, dy.$$

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