

Some Results of Chern-do Carmo-Kobayashi Type and the Length of Second Fundamental Form

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1. Introduction. Let M be an n -dimensional manifold immersed in an $(n + p)$ -dimensional Riemannian manifold N^{n+p} of constant sectional curvature c . Let \mathbf{h} be the second fundamental form of this immersion; it is a certain symmetric bilinear mapping $T_x \times T_x \rightarrow T_x^\perp$ for $x \in M$, where T_x is the tangent space of M at x and T_x^\perp the normal space of M at x . We denote by S the square of the length of \mathbf{h} . Let \mathbf{H} be the *mean curvature vector*, and $\langle \cdot, \cdot \rangle$ the scalar product of N^{n+p} . If there exists a function λ on M such that

$$(1.1) \quad \langle \mathbf{h}(\mathbf{X}, \mathbf{Y}), \mathbf{H} \rangle = \lambda \langle \mathbf{X}, \mathbf{Y} \rangle,$$

for all tangent vectors \mathbf{X}, \mathbf{Y} on M , then M is called a *pseudo-umbilical submanifold* of N^{n+p} . It is clear that $\lambda \geq 0$, and $\lambda^{1/2}$ is just the length of \mathbf{H} , called the *mean curvature*. If the mean curvature vector $\mathbf{H} = 0$ identically, then M is called a *minimal submanifold* of N^{n+p} . Every minimal submanifold of N^{n+p} is itself a pseudo-umbilical submanifold of N^{n+p} .

In general, let $S^q(r)$ denote a q -dimensional sphere in R^{q+1} with radius r . Let m and n be positive integers such that $m < n$ and let $M_{m;n-m} = S^m((m/n)^{1/2}) \times S^{n-m}((n - m/n)^{1/2})$. We imbed $M_{m;n-m}$ into $S^{n+1} = S^{n+1}(1)$ as follows. Let (\mathbf{u}, \mathbf{v}) be a point of $M_{m;n-m}$ where \mathbf{u} (resp. \mathbf{v}) is a vector in R^{m+1} (resp. R^{n-m+1}) of length $(m/n)^{1/2}$ (resp. $(n - m/n)^{1/2}$). We can consider (\mathbf{u}, \mathbf{v}) as a unit vector in $R^{n+2} = R^{m+1} \times R^{n-m+1}$. Then $M_{m;n-m}$ is a minimal submanifold of S^{n+1} satisfying $S = n$. Hence it is a pseudo-umbilical submanifold of S^{n+1} .

We shall now define the Veronese surface. Let (x, y, z) be the natural coordinate system in R^3 and $(u^1, u^2, u^3, u^4, u^5)$ the natural coordinate system in R^5 . We consider the mapping defined by

$$\begin{aligned} u^1 &= \frac{1}{3^{1/2}} yz, & u^2 &= \frac{1}{3^{1/2}} xy, & u^3 &= \frac{1}{3^{1/2}} xz, \\ u^4 &= \frac{1}{2(3^{1/2})} (x^2 - y^2), & u^5 &= \frac{1}{6} (x^2 + y^2 - 2z^2). \end{aligned}$$