

A Vietoris-Begle Theorem for Differentiable Maps

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The natural approach to a Vietoris-Begle theorem is to consider the Leray spectral sequence for the map. In order to identify the stalks of the Leray sheaf as cohomology modules of the fibres an assumption about the map is necessary. The usual assumption is for the map to be closed. With this assumption, Skljarenko [9] obtained a Vietoris-Begle theorem which allowed exceptional fibres over sets of small relative covering dimension. Working jointly and not using the Leray sequence, Professor Smith and myself [7] obtained a Vietoris-Begle theorem by supposing the map was a submersion (smooth surjection with no critical points). By combining these two ideas, this paper results in a Vietoris-Begle theorem in singular homology for a smooth surjective map with critical values on sets of small relative covering dimension.

Leray Spectral Sequence. If $f: X \rightarrow Y$ is a continuous map and if \mathcal{Q} is a sheaf on X , denote the Leray sheaf of f with compact support by $\mathcal{H}_c^*(f; \mathcal{Q})$. For other basic notation and terminology of sheaf theory, I refer to Bredon's text [1]. By restricting the range and domain of the map to finite dimensional manifolds the Leray spectral sequence can be given in terms of sheaf cohomology with compact support.

Theorem 1. *Let $f: X \rightarrow Y$ be a continuous map between finite dimensional manifolds, then for any sheaf \mathcal{Q} on X there is a spectral sequence where*

$$E_2^{p,q} = H_c^p(Y; \mathcal{H}_c^q(f; \mathcal{Q})) \Rightarrow H_c^{p+q}(X; \mathcal{Q})$$

with natural isomorphisms $\mathcal{H}_c^(f; \mathcal{Q}) \approx H^*(f^{-1}; \mathcal{Q})$.*

Proof. Since X and Y are finite dimensional they are locally compact. Thus the family of compact supports c is paracompactifying [1, pg 16]. Because Y is Hausdorff and c is paracompactifying, the fibres are c -taut in X [1, pg 52]. Noting that the family of compact supports on the Hausdorff manifolds X and Y are well adapted [1, pg 140], the result now follows from Theorem 4.6.1 of Bredon's text [1].

Dimension Theory. The covering dimension for a normal space X , denoted