

Exact Multiplicative Functionals in Duality

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1. Introduction. Let X and \hat{X} be standard Markov processes, with state space E , which are in duality relative to a σ -finite measure ξ . It was shown recently by Gettoor [5] that if M is an exact multiplicative functional of X , there is a uniquely determined multiplicative functional \hat{M} of \hat{X} having the property that the associated resolvents $\{V^\alpha\}$ and $\{\hat{V}^\alpha\}$ are in duality, or that in a sense, the subprocesses (X, M) and (\hat{X}, \hat{M}) are in duality also. The correspondence is shown to be multiplicative, and by using methods generalizing those of Revuz ([10], [11]) a number of properties of the dual relationship are established for natural multiplicative functionals.

It is the purpose of the present paper to extend the results of Gettoor [5] to the general case, and in so doing, we establish, as a major tool (Theorem 6.1), that every exact terminal time of X is a.s. of the form $\inf \{t > 0 : (X_{t-}, X_t) \in \Gamma\}$, where Γ is a Borel subset of $E \times E$. This generalizes a result of Blumenthal and Gettoor [1] which characterized the thin accessible terminal times of X as the hitting times of thin Borel sets.

In order to obtain these results, we study additive functionals of the processes which satisfy hypotheses a little weaker than the usual ones, but which are more appropriate for this situation as well as a number of others. These additive functionals are characterized, and by using the given characterization, it is found (Theorem 7.1) that the most general exact MF of X has the form

$$M_t = \prod_{s \leq t} [1 - F(X_{s-}, X_s)] 1_{[0, R)}(t) \exp \left\{ - \int_0^t f(X_u) dA_u \right\}$$

where R is an exact terminal time, $F \in (\mathcal{E} \times \mathcal{E})_+$, $f \in \mathcal{E}_+$, and A is an ordinary CAF of X . Using the general results on duality, one finds that the dual of M is obtained in the above form by replacing R by its dual, $F(x, y)$ by $\tilde{F}(x, y) = F(y, x)$, and A by its dual. The dual of $R = \inf \{t > 0 : (X_{t-}, X_t) \in \Gamma\}$ is $\hat{R} = \inf \{t > 0 : (\hat{X}_t, \hat{X}_{t-}) \in \Gamma\}$ and the dual of A is the CAF \hat{A} of \hat{X} having the same associated measure μ as A , so that $u_A^\alpha = U^\alpha \mu$ and $\hat{u}_A^\alpha = \mu \hat{U}^\alpha$ (see Theorems 8.7 and 8.8 for details.)

In the final section, we discuss a duality theory for the more general additive functionals introduced here, extending the results of [6] in a special case.