

Perturbations of Isometries on Hilbert Space

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1. Introduction. This paper is concerned with perturbations of isometries on infinite-dimensional Hilbert space. Our choice of studying isometries has been motivated by the fact that they (in particular, the shifts) provide many useful examples and counterexample to all parts of Hilbert space theory (see Halmos [1]). A brief description of our main result is as follows. Let H, H' be complex separable infinite-dimensional Hilbert spaces. Let S be an isometry on H and P be a bounded operator (throughout this paper, all operators are linear) on H which can be factored formally as the product B^*A of two bounded operators A, B on H to H' . Under certain assumptions on the behavior of A, B relative to S , we are able to construct a nonsingular operator $W(\mu)$ such that $(S + \mu P) = W(\mu)SW(\mu)^{-1}$ for all sufficiently small complex parameters μ . It follows that S and $S + \mu P$ are similar operators on H . This result has an interesting application to perturbation problem of the unilateral shift

$$S: (x_0, x_1, x_2, \dots) \rightarrow (0, x_0, x_1, \dots)$$

on $\ell^2(0, \infty)$. It is well known that the invariant subspaces of S have been characterized by Beurling, using function algebra representation [2]. In a paper of Duren [3], a study was made on certain "tridiagonal operators" on $\ell^2(0, \infty)$ whose lattices of invariant subspaces are isomorphic to that of S . Later Freeman [4] shows that for a large class of strictly lower-triangular matrices $P = (p_{mn})$, satisfying

$$|P| = \sum_{m,n=0}^{\infty} |p_{mn}| < \infty \quad \text{and} \quad p_{n+1,n} \neq -1,$$

the operators S and $S + P$ are similar to each other. In this paper we are able to establish the similarity between S and $S + P$ with only size restriction on P . More precisely, we show that $S + \mu P$ and S are similar operators on $\ell^p(0, \infty)$ ($1 \leq p \leq \infty$) for all infinite matrices $P = (p_{mn})$ with $|P| = \sum_{m,n=0}^{\infty} |p_{mn}| < \infty$ and for all complex parameters μ with $|\mu| < 1/2 |P|$.

Our approach is motivated by our previous success in treating perturbation of generators of continuous semi-groups [5] based on the theory of wave operators and smooth perturbation [6]. The idea is to construct a nonsingular bounded