

Slices and Potentials

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1. The operation of slicing an m dimensional normal current T in U by a locally Lipschitzian map f of U into \mathbf{R}^n , with $m \geq n$, was studied in [2, §3] and [3, §4.3] through relative differentiation of measures. It was shown that the slice $\langle T, f, y \rangle$ exists for \mathcal{L}^n almost all points y in \mathbf{R}^n , as an $m - n$ dimensional flat chain in U , and is \mathcal{L}^n summable with respect to y . Here it will be proved that the Hausdorff dimension of the exceptional set, consisting of those points y in \mathbf{R}^n for which $\langle T, f, y \rangle$ fails to exist, never exceeds $n - 1$. The new method combines techniques from potential theory and geometric measure theory, and yields also a residue formula for slices.

We use the terminology of [3], which contains glossaries of basic notation on pages 669 to 671.

2. In this section we discuss a few facts which we need from potential theory. Suppose M is the set of all (nonnegative) Radon measures over \mathbf{R}^n with compact support. Whenever $\mu \in M$ and $0 < s < \infty$ the potential P_μ^s is given by the formula

$$P_\mu^s(y) = \int |z - y|^{-s} d\mu z \quad \text{for } y \in \mathbf{R}^n,$$

where $0^{-s} = \infty$. Letting

$$M^s = M \cap \{\mu : P_\mu^s(y) \leq 1 \quad \text{for } y \in \text{spt } \mu\},$$

we define the s dimensional capacity first for open subsets V of \mathbf{R}^n as

$$\text{Cap}^s(V) = \sup \{\mu(V) : \mu \in M^s \quad \text{and} \quad \text{spt } \mu \subset V\},$$

then for arbitrary subsets A of \mathbf{R}^n as

$$\text{Cap}^s(A) = \inf \{\text{Cap}^s(V) : A \subset V \subset \mathbf{R}^n \quad \text{and} \quad V \text{ is open}\}.$$

The following nine propositions hold:

(1) *If V is an open subset of \mathbf{R}^n , then*

$$\text{Cap}^s(V) = \sup \{\mu(V) : \mu \in M^s\}.$$