

# *Invariant Subspaces of Abstract Multiplication Operators*

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§1. We want to describe a class of operators on a Banach space  $\mathfrak{B}$  whose members behave, in a sense, like multiplication operators, and consequently leave invariant a proper closed subspace of  $\mathfrak{B}$ —that is, they are *intransitive*. One of the sufficient conditions for an operator to be such an “abstract multiplication” is similar to an assumption made by Wermer [1] (and later by Ljubic and Macaev [2], Wolf [3]) who approached the invariant-subspace problem from a different (and rather more sophisticated) point of view. The approach indicated here is, perhaps, more intuitive, and we hope that the more general pattern which appears to emerge may be more than superficial.

Wermer considered operators whose deviation from being an isometry is limited. More precisely, he assumed that  $T$  and  $T^{-1}$  are both bounded, and that either

(A)  $\|T^n\| = O(e^{|\alpha|n})$ ,  $n = 0, \pm 1, \dots$ ,  $0 < \alpha < 1$ , and the spectrum of  $T$  contains at least two points, or

(B)  $\|T^n\| = O(|n|^k)$ , for some fixed  $k < \infty$ .

Such a  $T$  is *intransitive*. If  $\alpha = 0$  in (A), or  $k = 0$  in (B), then  $\|T^n\| \leq K$  for all  $n$ ; the space may then be renormed to make  $T$  an isometry, and intransitivity follows from a theorem of Godement [4].

The crucial estimate is that in (A); it is a reformulation of the requirement that

$$(1) \quad \sum_{n=-\infty}^{\infty} \frac{\log \|T^n\|}{1 + n^2} < \infty$$

which in turn plays an essential role in Wermer’s delicate and highly analytic proof.

We take as model of an intransitive operator not an isometry, but a normal operator on a Hilbert space. Because of the spectral representation theorem, such an operator is basically a multiplication by an  $L^\infty$ -function, acting on an  $L^2$ -space. By controlling the deviation of a more general operator from