

Some Banach Algebras Associated With Quotients of $L^1(\mathbb{R})$

YITZHAK KATZNELSON & CARRUTH McGEHEE

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1. Introduction. In this paper we continue the discussion begun in [11] of the algebras $B(E)$ associated with quotients $A(E)$.

Let A denote the Banach algebra of Fourier transforms g of elements \hat{g} of $L^1 = L^1(\mathbb{R})$:

$$g(x) = \int_{-\infty}^{\infty} \hat{g}(y)e^{-ixy} dy \quad \text{for } x \in \mathbb{R}.$$

In A we have pointwise multiplication and the norm $\|g\|_A = \|\hat{g}\|_{L^1}$. Let E always denote a compact subset of \mathbb{R} . Let $I(E)$ be the closed ideal in A made up of the functions in A that vanish on E ; let $A(E)$ denote the quotient algebra $A/I(E)$. Its elements may be thought of as the restrictions to E of functions in A . The norm is given by:

$$\|f\|_{A(E)} = \inf \{ \|g\|_A : g \in A \text{ and } g|_E = f \}.$$

This algebra is semisimple, and its maximal ideal space is E . Let $C(E)$ denote the algebra of all continuous functions on E , with the supremum norm. If $A(E) = C(E)$, then E is called a *Helson set*.

The Banach algebra $B(E)$ consists of the uniform limits on E of bounded sequences in $A(E)$, with norm

$$\|f\|_{B(E)} = \inf \{ \sup_n \|f_n\|_{A(E)} : \{f_n\}_{n=1}^{\infty} \subset A(E) \text{ and } \|f - f_n\|_{C(E)} \rightarrow 0 \}.$$

Evidently, for every E ,

$$A(E) \subseteq B(E) \subseteq C(E),$$

and each of the two inclusion maps is norm-decreasing. For a Helson set, the three algebras are the same and the three norms are equivalent. Various conditions on E are known that imply $A(E) = B(E)$; for example, if every portion of E supports a nonzero measure μ such that $\lim_{|y| \rightarrow \infty} \hat{\mu}(y) = 0$, then $A(E) =$