

The Product of Two Involutions in the Unitary Group of a Hermitian Form

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In this paper we denote by V a finite dimensional vector space over a field K . By J we denote an automorphism of K such that $J^2 = 1$. In particular, one can take $J = 1$. By $\Phi: V \times V \rightarrow K$ we denote a non-degenerate hermitian form. Thus we have

$$\Phi(\lambda x, \mu y) = \lambda \mu^J \Phi(x, y), \quad \Phi(x, y) = \Phi(y, x)^J$$

for all $\lambda, \mu \in K$ and $x, y \in V$. If $J = 1$, then Φ is in fact a non-degenerate symmetric bilinear form.

We denote by $U(\Phi)$ the unitary group of the form Φ . If A is a linear transformation of V , then A^* will denote the adjoint of A with respect to Φ . Hence we have $\Phi(xA, y) = \Phi(x, yA^*)$ for all $x, y \in V$. A linear transformation A of V is an *involution* if A^2 is the identity transformation I .

Let X be an indeterminate over K . If

$$f(X) = \sum_{i=0}^m \alpha_i X^i \in K[X],$$

we define

$$f^J(X) = \sum_{i=0}^m \alpha_i^J X^i.$$

If, in addition $\alpha_0 \neq 0$ and $\alpha_m = 1$, then we put

$$\tilde{f}(X) = \alpha_0^{-1} X^m f(X^{-1}).$$

The object of this paper is to prove the following.

Theorem 1. *Let $A \in U(\Phi)$ and assume that either*

- (i) $J = 1$, or
- (ii) $J \neq 1$, A is similar to A^* and $f \neq \tilde{f}^J$ for every monic irreducible factor of the minimal polynomial of A .

Then A is a product of two unitary involutions.