

# *Heat Transfer by Conduction and Radiation*

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There are a number of physical situations in which heat transfer occurs by both conduction and radiation. Heat transfer in molten glass is an easy one to visualize. Another is the effect of a ceramic coating on the heat transfer from a metal subsurface. Although there are quite a few studies which develop suitable mathematical models and there are a number of studies aimed at computing solutions, there seems to be no analysis of the basic questions of existence and uniqueness. Among the literature we mention [1, 3-5].

The physical problem which concerns us is relatively simple in structure but typical of those in the literature. We are concerned with one dimensional energy transfer through a gray medium bounded by two infinite, parallel, gray, diffuse surfaces. The absorption and scattering coefficients and the thermal conductivity are independent of temperature and steady state conditions are assumed to prevail. This problem is of physical interest despite its simplifying assumptions and already leads to a rather complex mathematical problem. We use below the dimensionless formulation of [3, chap. 9]. In our analysis we prove the existence of physical solutions for all physically meaningful parameter values. For a restricted range of parameters the solutions are shown to be unique. The existence results are constructive and may be made the basis of computer codes for approximating solutions. Lastly we examine the validity of an approximate solution arising from physical considerations.

The mathematical problem is

$$(1) \quad N \frac{d^2 \theta(\tau)}{d\tau^2} = (1 - \omega_0)[\theta^4(\tau) - \tfrac{1}{4}\eta(\tau)]$$

$$(2) \quad \theta(0) = 1, \quad \theta(\tau_0) = \theta_2,$$

where

$$(3) \quad \eta(\tau) = 2\chi_1 E_2(\tau) + 2\chi_2 E_2(\tau_0 - \tau) \\ + 2 \int_0^{\tau_0} \left[ (1 - \omega_0) \theta^4(t) + \frac{\omega_0}{4} \eta(t) \right] E_1(|\tau - t|) dt$$