

The Regularity of Minimal Surfaces on the Movable Parts of Their Boundaries

JOHANNES C. C. NITSCHÉ

Communicated by J. MOSER

A general version of Plateau's problem may be formulated as follows:

Determine a surface of least area which has a prescribed topological type and which is bounded by a configuration of several fixed contours, supporting surfaces, and movable threads of given lengths.

Soap laminae spanning frames with movable contours—experimentally realized by hairs or thin threads—have been described, among other places, in [2; p. 48], [3; p. 174], [8; p. 584], [15; pp. 7, 8, 20]. As is well known, the existence of a solution hinges on certain metric-geometric conditions which must be satisfied by the bounding configuration. Otherwise the minimizing sequences on which the existence proof is based do not converge or, if they do, converge only to degenerate limit structures—disconnected surfaces of lower topological types or partly one-dimensional formations. There are interesting examples illustrating these contingencies. In a number of cases, however, existence proofs are feasible by an extension of the classical methods, leading to a solution surface $S = \{x = x(u, v); (u, v) \in P\}$ whose position vector appears as function of isothermal parameters. Here P is a domain containing those boundary components which correspond to the fixed and movable contours.

Today the boundary behavior of the solution surface is fully studied for the fixed contours and is satisfactorily understood for the free boundaries, *i.e.*, the boundary portions on supporting manifolds. Almost nothing, however, is known so far for the movable contours. In [11] a theorem had been proved provided certain regularity assumptions dictated by intuition could be taken for granted:

The movable parts of the boundary must be curves of constant space curvature. On the solution surface itself they are asymptotic lines of constant geodesic curvature.

As was pointed out in [11], there is an obvious and challenging connection to the isoperimetric problem on surfaces. If a movable contour were to vary