

The Law of the Iterated Logarithm for Multiplicative Systems

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G. Alexits in [1] introduced the following concepts:

Definition 1. A sequence ξ_1, ξ_2, \dots of random variables is called a *multiplicative system* if

$$\mathbf{E}(\xi_{i_1} \xi_{i_2} \cdots \xi_{i_n}) = 0 \quad (i_1 < i_2 < \cdots < i_n; n = 1, 2, \dots).$$

Definition 2. A sequence ξ_1, ξ_2, \dots of random variables is called an *equi-normed strongly multiplicative system (ESMS)* if

$$\mathbf{E}(\xi_i) = 0, \quad \mathbf{E}(\xi_i^2) = 1 \quad (i = 1, 2, \dots)$$

and

$$\mathbf{E}(\xi_{i_1}^{r_1} \xi_{i_2}^{r_2} \cdots \xi_{i_n}^{r_n}) = \mathbf{E}(\xi_{i_1}^{r_1}) \mathbf{E}(\xi_{i_2}^{r_2}) \cdots \mathbf{E}(\xi_{i_n}^{r_n}) \quad (i_1 < i_2 < \cdots < i_n; n = 1, 2, \dots)$$

where $r_i (i = 1, 2, \dots, n)$ can be equal to 1 or 2.

The law of the iterated logarithm for these systems was studied in [2], [3], [4], [5], [6], [7]. All of these papers were only interested in the upper part of the law of the iterated logarithm, *i.e.* they proved under different conditions that

$$\mathbf{P} \left\{ \overline{\lim}_{n \rightarrow \infty} \frac{\xi_1 + \xi_2 + \cdots + \xi_n}{(2n \log \log n)^{1/2}} \leq C \right\} = 1$$

for certain positive C .

Probably the best results in this direction are due to Shigeru Takahashi [7] and V. F. Gaposhkin [6] who have proved the following two theorems:

Theorem A. [7] *Let ξ_1, ξ_2, \dots be a uniformly bounded multiplicative system for which*

$$\mathbf{E}(\xi_i^2) = 1, \quad \mathbf{E}(\xi_i^2 \xi_j^2) = 1 \quad (i = 1, 2, \dots; j = 1, 2, \dots; i \neq j).$$

Then