

Commutators and Scattering Theory II.

A Class of One Body Problems

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Communicated by R. PHILLIPS

1. Introduction. A quantum mechanical particle in \mathbf{R}^3 interacting with a potential $V(x)$ which approaches zero as $|x| \rightarrow \infty$ is expected either to be bound with negative energy or to be scattered, behaving as $t \rightarrow \pm \infty$ like a free particle. This dichotomy would be reflected mathematically by a representation of the Hilbert space

$$(1.1) \quad \mathcal{K} = \mathcal{L}^2(\mathbf{R}^3) = \mathcal{K}_b \oplus \mathcal{R}(\Omega_{\pm})$$

where \mathcal{K}_b is the subspace spanned by all negative energy eigenvectors of the Hamiltonian operator $H = H_0 + V$ ($H_0 = -\Delta$), and $\mathcal{R}(\Omega_{\pm})$ is the range of the wave operator

$$(1.2) \quad \Omega_{\pm}(H, H_0) = s\text{-}\lim_{t \rightarrow \pm\infty} e^{iHt} e^{-iH_0 t}.$$

(The statement (1.1) is meant to imply $\mathcal{R}(\Omega_+) = \mathcal{R}(\Omega_-)$.)

The problem of proving (1.1) can be divided into four parts:

(1.3) Existence of the limit (1.2).

(1.4) Absence of positive eigenvalues for H .

(1.5) $\mathcal{R}(\Omega_{\pm}) = \mathcal{K}_{ac}$ (the range of the absolutely continuous part of $E(\cdot)$, the spectral family of H .)

(1.6) $\mathcal{K}_{sc} = \{0\}$ (\mathcal{K}_{sc} is the range of the singular continuous part of $E(\cdot)$.)

For $V(x) \leq C|x|^{-\gamma}$, $\gamma > 1$, this program has been carried out (see [6]) except for one gap: for $\gamma < 5/4$ (1.6) has not been proved, although a proof of (1.6) under more general circumstances was announced by S. Agmon at the 1970 I. C. M. in Nice. For $0 < \gamma \leq 1$ the situation is different. Since the wave operators fail to exist in general [4], problems (1.3) and (1.5) must be reformulated. Also, positive energy eigenvalues may actually exist if V oscillates badly. Problem (1.4) has been solved for potentials which do not oscillate too badly