

# *Some Limit Theorems for Stochastic Equations and Applications*

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**1. Introduction.** Our aim in this paper is to provide mathematical justification and explore further the formal method of analyzing stochastic equations introduced in [14] and previously, in a different manner, in [11, 12]. This method amounts to an adaptation of the two-variable method [2] to stochastic problems. When applicable, it yields very good results. An explanation for this success is provided by the fact that the result is a natural generalization of the classical Central Limit Theorem [4] to the present context of stochastic equations. The fundamental investigations of Stratonovich [17, 18] and Hashminskii [7] have already established this generalized Central Limit Theorem for a broad class of stochastic ordinary differential equations. The analytical approach we follow here is operator theoretical. We have attempted to make the formal considerations of [14] precise without deviating too much from intuitively appealing arguments.

In Section 2 we state and prove a limit theorem for an abstract stochastic differential equation. We assume here that the stochastic structure of the problem is quite simple and this permits the use of rather standard arguments. Undoubtedly the result is true for considerably more general stochastic structures.

In Section 3 we use the result of Section 2 to study the limiting behavior of random products of random matrices. We do this in the context of Lie groups, for this seems to be the appropriate setting. Theorem 2 is similar to the result of Wehn [22, 6] but the hypotheses of the two results are not commensurable. We close Section 2 with a simple application of Theorem 2 to the group of rotations about a fixed point in 3 dimensions.

Section 4 contains a generalization of Theorem 1 of Section 2 to the case where the abstract stochastic equation depends explicitly on time in a specific way. The problems considered in [14] are all of this type. We have found it necessary however to specialize the stochastic structure of the problem even further. This is a weakness of the method employed rather than an essential feature.