

# On the Swirling Flow Problem

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*Communicated by* DAVID GILBARG

1. The object of this note is to give a simple proof of the following:

**Theorem (\*).** *There exists a solution of the singular (autonomous) boundary value problem*

$$(1.1) \quad f''' + ff'' + \beta(g^2 - \omega^2 - f'^2) = 0,$$

$$(1.2) \quad g'' + fg' - 2bf'g = 0,$$

$$(1.3) \quad f(0) = f_0, f'(0) = f'_0 \quad \text{and} \quad g(0) = g_0,$$

$$(1.4) \quad f'(\infty) = 0 \quad \text{and} \quad g(\infty) = \omega,$$

*under the assumption*

$$(1.5) \quad 0 < \beta < 2b \quad \text{and} \quad g_0 > 0, \quad \omega > 0.$$

Existence has been proved by McLeod [7] in the case  $b = \beta = 1/2$  and  $f'_0 = 0$ , which corresponds to the existence of a solution of the Navier–Stokes equations for an axially symmetric flow above an infinite rotating disc (where, if  $\nu = 1$  and  $(r, \theta, t)$  are cylindrical coordinates, then the velocity in these coordinates is  $(rf'(t), rg(t), -2f(t))$ ; cf. Serrin [10], pp. 29–30). For references and other existence and non-existence theorems related to (1.1)–(1.4), see Watson [11], McLeod [6], and Hartman [3].

It is not clear that McLeod's proof [7] is valid under the conditions (1.5). The main point of this note, however, is the brevity and simplicity of the proof compared to that of McLeod [7] (or Watson [11] or Hartman [3]). It will not involve *a priori* asymptotic formulas for the solutions (cf. [5], [3]), nor a separation of cases  $f_0 \leq 0$  and  $f_0 > 0$ , nor singular integral equations. Some of the basic *a priori* estimates below, however, are obtained by methods suggested by McLeod [7]. We first prove existence for a non-singular boundary value problem on a finite interval  $0 \leq t \leq T$ , namely, (1.1)–(1.3) and assigned  $f'(T)$ ,  $g(T)$ ; take a limit as  $T \rightarrow \infty$ ; and finally, verify the conditions (1.4) at  $t = \infty$ . The treatment below of the boundary value problem on  $[0, T]$  follows the methods of [1]; cf. [2], pp. 428–434. Other applications of these methods to