

# *Nerve Axon Equations: 1 Linear Approximations*

JOHN W. EVANS

*Communicated by* MURRAY ROSENBLATT

**1. Introduction.** The theory of a cable consisting of a resistive core surrounded by a membrane offering capacitance and variable resistance to ionic currents is of central importance in neurophysiology. The equations of Hodgkin and Huxley [9] describing the dynamics of a nerve impulse are based on this theory as are the models of whole nerve cell electrophysiology of Rall [11]. A brief derivation of the class of equations follows.

If  $r$  is the core resistance of the cable per unit length and  $V(x, t)$  is the core or intracellular voltage at distance  $x$  and time  $t$ , then  $(1/r)(\partial V/\partial x)$  is the current down the core, and by Kirchoff's law  $(1/r)(\partial^2 V/\partial x^2)$  is the current across the membrane. Assuming that the outside resistance is negligible so that the voltage outside the cable is constant, this transmembrane current has a capacitive component  $c(\partial V/\partial t)$  where  $c$  is the membrane capacitance per unit length and a resistive component  $i(V, W^1, \dots, W^n)$  which depends on  $V$  and on a number of auxiliary state variables  $W^1(x, t), \dots, W^n(x, t)$  which determine the resistances in specific ionic pathways (sodium, potassium etc.). The auxiliary variables obey equations of the type  $(\partial W^i/\partial t)(x, t) = g^i(V, W^1, \dots, W^n)$  at each point  $x$  along the cable so that communication between different locations is effected solely through voltage changes. Rescaling distance so as to eliminate constants and letting  $W^0 = V$  and  $g^0 = -i/c$  we obtain

$$(1) \quad \begin{aligned} \frac{\partial W^0}{\partial t} - \frac{\partial^2 W^0}{\partial x^2} &= g^0(W^0, \dots, W^n) \\ \frac{\partial W^i}{\partial t} &= g^i(W^0, \dots, W^n), \quad i = 1, \dots, n. \end{aligned}$$

This general form given by FitzHugh [8] includes the equations of Hodgkin and Huxley [9], FitzHugh [7], Rall [11] and Nagumo [10]. An essential feature of these models is a stable or resting state where  $g^0 = \dots = g^n = 0$ . We take this to be at  $W^0 = \dots = W^n = 0$ .