

Branching Markov Processes in a Random Environment

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0. Introduction. Recently, Smith and Wilkinson [6] and Athreya and Karlin [1] have investigated a generalized branching model: a branching process in a random environment. They focused their attention on the discrete time model. The model is briefly described as follows. Let $\langle \zeta_n \rangle_{n=0}^\infty$ be a sequence of “environmental” elements on some probability space. Corresponding to each ζ_n we have an associated probability generating function $h_{\zeta_n}(s)$. This gives us then a random sequence of probability generating functions $\langle h_{\zeta_n} \rangle_{n=0}^\infty$. For each fixed realization of the environment the population process evolves in the following manner. The particles alive in the n^{th} generation create progeny independently of one another according to h_{ζ_n} . The totality of the new-born progeny make up the $(n + 1)^{\text{st}}$ generation. In other words, the population process conditioned on the environment evolves as a temporally non-homogeneous branching process.

Smith and Wilkinson originally formulated the model and considered the extinction problem under the assumption that the ζ_n were independent and identically distributed. Athreya and Karlin considered a more general case in which $\langle \zeta_n \rangle_{n \geq 0}$ formed a stationary ergodic process. They showed that most of the classical results for a Galton–Watson process have their counterparts in this set-up.

In this paper we extend the notion of a branching process in a random environment in two directions. Firstly, we consider a continuous time model on any “nice” topological state space. (N. Kaplan [4] has investigated a continuous time version for the special case of a single-type particle.) Secondly, not only do we allow the branching kernel to depend upon the environmental process, but also the parameters which govern the underlying motion of the particles. In particular, the rate of births may depend upon the environment. The model \mathfrak{z} is formulated so that the conditioned process \mathfrak{z}^w (*i.e.*, the branching process conditioned on the environment w) is a branching Markov process (bmp) in the sense of Ikeda, Nagasawa and Watanabe [3]. Consequently, for the condi-