

# *Monotone Methods in Nonlinear Elliptic and Parabolic Boundary Value Problems*

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**1. Introduction.** In this paper we discuss the use of monotone iteration schemes in the construction of solutions of nonlinear elliptic boundary value problems of the type

$$(1.1) \quad \begin{aligned} Lu + f(x, u) &= 0 \quad \text{in } D, \\ Bu &= g \quad \text{on } \partial D \end{aligned}$$

where  $L$  is a second order uniformly elliptic operator:

$$L = \sum_{i,j=1}^n a_{ij} \frac{\partial^2 u}{\partial x^i \partial x^j} + \sum_{i=1}^n b_i \frac{\partial u}{\partial x^i}, \quad x = (x^1, \dots, x^n)$$

and  $B$  is one of the boundary operators

$$Bu = u,$$

or

$$Bu = \frac{\partial u}{\partial \nu} + \beta(x)u, \quad x \in \partial D.$$

Here  $\partial/\partial \nu$  denotes the outward conormal derivative, and we assume  $\beta \geq 0$  everywhere on the boundary,  $\partial D$ .

The coefficients of  $L$  are assumed to be smooth (say Hölder continuous) and the matrix  $a_{ij}$  is to be uniformly positive definite on  $\bar{D}$ . Any undifferentiated terms in  $u$  may be grouped with the term  $f(x, u)$ , including linear terms. We assume the boundary of  $D$  is smooth, say  $C^{2+\alpha}$ , though this condition could be relaxed somewhat.  $f(x, u)$  is assumed smooth throughout this paper, say  $f$  is  $C^1$  in  $u$  and Hölder continuous in  $x$ .

Such nonlinear boundary value problems arise in numerous applications (see references) and in fact the present paper was motivated by D. S. Cohen's work [4] on problems arising in chemical reactor theory. See also [12], [13].