

A Topological Three Pole Theorem

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1. Following the manifestation by Teichmüller of the essential role played by quadratic differentials in many important questions in Function Theory, various authors undertook a more detailed study of these entities. Schaeffer and Spencer gave an extended discussion of the local structure of the trajectories [9; Chap. III], verifying the basic results which had been stated by Teichmüller with little indication of proof. Actually their statements are given for hyperelliptic quadratic differentials (meromorphic quadratic differentials on the sphere), but naturally the results are valid in general. They also attempted to analyze the global structure at least in the case of hyperelliptic differentials. In their approach an essential role was played by the proof that all open trajectories were non-recurrent in the sense that no point of such a trajectory can be a limiting accumulation point of points of the trajectory. They succeeded in proving this for hyperelliptic differentials with one pole, certain hyperelliptic differentials with two poles and for a particular hyperelliptic differential with three poles, and hoped to be able to carry out this process in general. The author early recognized that recurrent trajectories can occur, in fact that for the differential $c[(z - z_1)(z - z_2)(z - z_3)(z - z_4)]^{-1} dz^2$ for all but a countable number of choices of the argument of c each trajectory will be everywhere dense on the sphere. More general constructions given in [2] can be used to show that in the case of finite Riemann surfaces of positive genus, recurrent trajectories can occur, and in the case of plane domains if the total number of distinct poles and boundary components is at least four, recurrent trajectories can occur for positive quadratic differentials.

Actually, as far as applications are concerned the presence of recurrent trajectories frequently need cause no difficulty. This can be deduced from the work of Teichmüller [10] in the case of quadratic differentials with only finite critical points. The author's proof of the General Coefficient Theorem [5, 6] shows that this is true in the more general situation. However there is considerable intrinsic interest in the question of recurrence. With the proof of the Basic Structure Theorem [6] by the author, it was not difficult to prove that for hyperelliptic quadratic differentials with at most three distinct poles, recurrent