

# *Stability of Kleinian Groups*

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**§1. Introduction.** In this paper we begin the study of the local properties of the affine algebraic variety of homomorphisms of a finitely generated Kleinian group  $\Gamma$  into the Möbius group  $G$ . We are following a program charted by André Weil in his beautiful paper [22]. Indeed, we use the techniques and a key lemma from [22]. For the sake of completeness we have reproduced some of Weil's arguments in this paper.

Our main result (Theorem 8.4) is a sufficient condition for quasiconformal stability of  $\Gamma$ . It transforms a non-linear problem into a linear one. However, the linear problem is also quite difficult and we do not, at present, have a complete answer to this linear version of the question that motivated this investigation. We are trying to determine when  $\Gamma$  is quasiconformally stable; that is, when every sufficiently small admissible (to be defined in §2) homomorphism of  $\Gamma$  into  $G$  is induced by a global quasiconformal automorphism of the complex sphere  $\hat{\mathbb{C}}$ . In quite an obvious way one identifies the "tangent space" to the variety of homomorphisms of  $\Gamma$  into  $G$  at the identity with a subset of the cocycles for  $\Gamma$  with coefficients in the Lie algebra  $\mathfrak{g}$  of  $G$ . It turns out that the cohomology group  $H^1(\Gamma, \mathfrak{g})$  is canonically isomorphic to the Eichler cohomology group  $H^1(\Gamma, \Pi)$ ,  $\Pi =$  quadratic polynomials. These groups have been studied by Ahlfors [3] and Kra [10], [11] among others. A sufficient condition for stability is that every parabolic cohomology class in  $H^1(\Gamma, \Pi)$  be a Bers class.

Our results should be contrasted with those of Bers [8], Marden [16], Maskit [19], and Mostow [20]. Bers [8] obtains a different, less explicit, sufficient condition for stability. Marden [16] works with finitely generated Kleinian groups that have finitely sided fundamental polyhedra and contain no elements of finite order. He studies stability and geometric properties of such groups. Maskit [19] shows that all "constructible" groups with invariant components and/or groups satisfying other (perhaps equivalent) "niceness" conditions are stable. Mostow [20] shows that every discrete subgroup  $\Gamma$  of  $G$  of the first kind ( $G$  may be viewed as a group of conformal self-mappings of the unit ball  $B$  in 3-space; that is,  $B$  is hyperbolic 3-space and *first kind* means that  $\Gamma$  is discrete (and hence discontinuous on  $B$ ) but does not act discontinuously at any point