

Quantities Related to Strictly Singular Operators

MARTIN SCHECHTER

Communicated by RALPH PHILLIPS

1. Introduction. In a sense, this paper is a sequel to [1], where we studied seminorms which measured the “non-compactness” of operators. If we had followed the terminology used there, we would have called the present paper “Measures of non-strict-singularity.” We introduce four “measures” as follows: For an operator $A \in B(X, Y)$ we set

$$\begin{aligned} \Gamma(A) &= \operatorname{glb}_M \|A|_M\|, \\ \Delta(A) &= \operatorname{lub}_M \operatorname{glb}_{N \subset M} \|A|_N\|, \\ \tau(A) &= \operatorname{lub}_M \operatorname{glb}_{\substack{x \in M \\ \|x\|=1}} \|Ax\|, \\ \nu(A) &= \operatorname{lub}_{\operatorname{codim} M < \infty} \operatorname{glb}_{\substack{x \in M \\ \|x\|=1}} \|Ax\|, \end{aligned}$$

where M, N represent infinite dimensional subspaces of X and $A|_M$ denotes the restriction of A to M . We prove the following relationships among these quantities:

- (1.1) $\nu(A) \leq \tau(A) \leq \Delta(A)$ (Theorem 2.6).
- (1.2) $\nu(A) \leq \Gamma(A) \leq \Delta(A)$ (Theorem 2.19).
- (1.3) $\Gamma(A + B) \leq \Gamma(A) + \Delta(B)$ (Theorem 2.1).
- (1.4) $\Delta(A + B) \leq \Delta(A) + \Delta(B)$ (Theorem 2.2).
- (1.5) $\Gamma(AB) \leq \Delta(A)\Gamma(B)$ (Theorem 2.3).
- (1.6) $\Delta(AB) \leq \Delta(A)\Delta(B)$ (Corollary 2.5).
- (1.7) $\Gamma(A) = \operatorname{glb}_Z \operatorname{glb}_{B \in B(Z, X)} \frac{\|AB\|}{\tau(B)}$ (Theorem 2.8).