

# *Generic Homotopies of Immersions*

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**1. Introduction.** Properties of an immersed manifold which are invariant under regular homotopy may be studied by analyzing immersions that lie in general position. The self-intersection pattern of such “generic” immersions frequently allows combinatorial characterization of these properties. For example, by the Whitney–Graustein theorem [14, p. 279], the tangent winding number of a plane immersed circle is a complete invariant up to regular homotopy. In the same fundamental paper, Whitney showed the normal immersions to be a dense open subset of  $C^1(S^1, R^2)$  in the uniform  $C^1$ -topology. The tangent winding number of a normal immersion can, in turn, be easily computed from the sequence of signed self-intersections. This intersection sequence has been formalized and extensively studied by C. J. Titus [12] and others [3, 7]. In particular, the intersection sequence is a complete topological invariant for normal plane immersed circles [13]. Hence, a regular homotopy passing only through normal stages cannot alter topological properties of the immersions, such as bounding a light-open image of the disk. To study properties that may be altered by a regular homotopy, Titus proposed the investigation of regular homotopies whose stages display, in addition to the transverse double point self-intersections of normal immersions, also isolated instances of convex double point self tangencies and transverse triple points. Bounding properties of certain of these Titus homotopies were studied in [4].

In the present paper we adapt Mather’s [9, p. 310] brilliant formulation of Thom’s transversality theorem for multijets (Proposition 2.10) to residual classes of parametrized mappings of manifolds (Proposition 3.3), such as generic classes of regular homotopies (Proposition 3.4). In particular (Section 4), Titus’ homotopies form a dense-open (generic) subset of the  $C^2$ -regular homotopies topologized in the Whitney  $C^2$ -topology.

**1.1 Remark.** We have chosen Mather’s transversality theory over other versions based on Morlet’s lemma [10, p. 6-03; 1, p. 471; 2, p. 197; 9, p. 310] for its greater simplicity. It is Morlet’s lemma that really simplifies the original proofs [11, p. 61; 6, p. 21]. We hope that our review of Mather’s theory (Section 2)