

Regularity of Solutions to Boundary Value Problems for First Order Systems

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§0. Introduction. This paper is concerned with existence, uniqueness, and regularity of solutions to boundary value problems for systems of partial differential equations for which one is able to establish certain L^2 *a priori* inequalities. The prototype of all such *a priori* estimates is

$$(0.1) \quad \|u\|_{s_1}^2 \leq C_1 \|Lu\|_{s_2}^2 + C_2 \|u\|_{s_3}^2, \quad u \in B,$$

where L is a $p \times p$ system of partial differential operators of order m in a domain Ω in R^n , $\|\cdot\|_{s_i}$ is a suitable Sobolev norm roughly measuring the square-integrability in Ω of derivatives of order $\leq s_i$ of $u(x) = (u_1(x), \dots, u_p(x))$, $s_3 < s_1 \leq s_2 + m$, and $u \in B$ will mean $u \in C^\infty(\bar{\Omega})$ satisfies certain (homogeneous or inhomogeneous) boundary conditions.

Morrey and Nirenberg consider the elliptic case with zero Dirichlet boundary conditions in [15] where one may take $s_1 = m + s_2$ and their result is a local real analytic one for solutions u to $Lu = f$ (assuming the boundary of Ω and the coefficients of L are real analytic).

Hörmander [10] and others consider the “subelliptic” case where $m + s_2 - 1 < s_1 < m + s_2$ and obtain hypoellipticity results (*i.e.*, if $Lu = f$, then u is C^∞ where f is).

The author has recently obtained a somewhat stronger result for the subelliptic case, namely that u belongs to the $(s_1 + 1 - s_2 - m)^{-1}$ -th Gevrey class where f does, and this result will be published elsewhere.

In the present paper we consider the case where $s_1 = s_2 + m - 1$ and the constant C_1 is small. Kohn and Nirenberg in [12] assume either that (0.1) is valid with all $C_1 > 0$ and $C_2 = C_2(C_1)$ or that (0.1) is valid with sufficiently small C_1 and also with $C_2 = 0$ and some C_1 , and they treat a much more general case than we do. In our simple setting their results are less sharp than ours.

In particular we take $m = 1$ although our results may clearly be extended to higher order systems. And our results are of the form: if $Lu = f$ with f in some