

Operator Algebras of Finite Strict Multiplicity

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Introduction. A subalgebra \mathbf{A} of the algebra $\mathbf{L}(\mathbf{X})$ of all (bounded linear) operators from the complex Banach space \mathbf{X} into itself is said to be “*of finite strict multiplicity*” (\mathbf{A} is an a.f.s.m.) if there exists a finite set $\Gamma = \{x_1, \dots, x_n\} \subset \mathbf{X}$ such that

$$(1) \quad \mathbf{X} = \text{linear span } \{Ax_i : A \in \mathbf{A}; \quad j = 1, 2, \dots, n\} = \mathbf{A}(\Gamma).$$

Here, and in what follows, *algebra* means *strongly closed* subalgebra of $\mathbf{L}(\mathbf{X})$ containing the identity I of $\mathbf{L}(\mathbf{X})$; *operator* and *subspace* mean *everywhere defined bounded linear transformation* (from \mathbf{X} into \mathbf{X}) and *closed linear manifold*, respectively.

This article attempts to be an introductory survey of the algebras of finite strict multiplicity; thus, the sections are not necessarily connected.

Results include the study of the properties of an algebra \mathbf{A} whose commutant is an a.f.s.m., the spectrum of the operators in \mathbf{A} and their adjoints, *etc*; in particular, it is shown that the uniform and the strong operator topologies coincide on \mathbf{A} . The case when \mathbf{A} is generated by the rational functions in a single operator is especially considered, and the *third section* is completely devoted to bilateral and unilateral weighted shifts which generate a separated s.c.a. It is also shown that every closed densely defined linear map commuting with a set of generators of an a.f.s.m. is necessarily bounded.

The last two sections deal with general commutative operator algebras, open problems and examples that illustrate that some of the results of the article (and related papers) are “sharp.”

In many aspects, this article is a sequel of [9].

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§1. Preliminary results. The definition of “multiplicity of an operator” given in [21] can be extended to arbitrary algebras as follows:

$$(2) \quad \mu(\mathbf{A}) = \text{multiplicity of } \mathbf{A} = \inf \{\text{card } (\Gamma); \mathbf{X} = \text{closure } \mathbf{A}(\Gamma)\} \quad (\text{where } \Gamma \text{ runs over all subsets of } \mathbf{X} \text{ and } \mathbf{A}(\Gamma) \text{ is interpreted as in (1)}).$$