

Periodic Orbits of the Second Kind

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Introduction. In Volume 1, Chapter 3 of his “*Les Méthodes Nouvelles de la Mécanique Céleste*”, Poincaré discusses periodic orbits. In particular, in Sections 42–48 he discusses periodic orbits of the second and third kind. At one point in his discussion Poincaré utilizes a result about doubly periodic functions that later P. Stäckel [4] showed to be false. Wintner [7, p. 47] also found fault with this work. Thus it is no surprise that mathematicians and astronomers have not utilized these sections although they pertain to a very important problem.

In recent years, starting with a paper of R. Arenstorf [1], mathematicians have returned to these problems, and have devised new methods to prove special cases of Poincaré’s results.

In the present paper we prove the existence of periodic orbits of the second kind for the three body problem, when the unperturbed primaries do not collide. Our analysis easily extends to periodic orbits of the third kind.

Our proof is based on a recent paper of A. Weinstein [6].

We begin with some notation and a statement of the problem.

Three body problem. The three body problems involve the notion of three points p_1 , p_2 , p_3 of masses m_1 , m_2 , m_3 respectively which attract each other according to the Newtonian laws of gravitation. For the purposes of the present paper we will assume that the three bodies move in the fixed x, y plane.

By utilizing the conservation of linear momentum we may assume that the center of mass is at rest at the center of coordinates. Hence to describe the motion it is sufficient to describe the relative position of two of the bodies. Let (x_1, x_2) be the position vector of p_2 with reference to p_1 , and let (x_3, x_4) be the position vector of p_3 with respect to the center of mass of p_1 and p_2 . Then as developed, for example, in Sternberg [5, Chapter IV, Section 8], with a normalization introduced by Jeffreys and Moser [2], the equations of motion are Hamiltonian in form

$$\frac{dx_i}{dt} = \frac{\partial H}{\partial x_i}, \quad \frac{dX_i}{dt} = -\frac{\partial H}{\partial x_i}, \quad i = 1, \dots, 4,$$