

Singular Integrals whose Kernels Involve Certain Sturm-Liouville Functions, II

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1. Introduction. Here, as in [12], we study the behaviour for large positive ν of the singular integral

$$\int_b^\infty f(t)w(\nu, t-x) dt,$$

where $b < x \leq b-a$ for some $a < 0$, a and b are finite, and, for each $\nu > 0$, $w(\nu, t)$ is the solution of

$$d^2w/dt^2 = [\nu^2t + q(t)]w,$$

on the interval $[a, \infty)$, which satisfies

$$(1.1) \quad \lim_{t \rightarrow \infty} w(\nu, t)t^{1/4} \exp\left(\frac{2}{3}\nu t^{3/2}\right) = \frac{1}{2}\pi^{-1/2}\nu^{1/2}.$$

(The assumption in [12, §4] that $a \leq b$ is unnecessary and is not used in any of the proofs.) To ensure that, for each $\nu > 0$, the condition (1.1) determines $w(\nu, t)$ uniquely, we suppose that q is continuous on $[a, \infty)$ and that

$$\int_0^\infty t^{-1/2} |q(t)| dt$$

exists.

In [12], we considered the convergence, as $\nu \rightarrow \infty$, of the foregoing singular integral; here, we consider its Cesàro summability.

The principal result in [12] is that, under appropriate hypotheses on f ,

$$(1.2) \quad \lim_{\nu \rightarrow \infty} \int_b^\infty f(t)w(\nu, t-x) dt = \frac{2}{3}f(x-) + \frac{1}{3}f(x+).$$

In particular, we have the following result, as a special case of [12, p. 865, Theorem 4.1].