

# *A Variant of the Lusternik-Schnirelman Theory*

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*Communicated by* DAVID GILBARG

**1. Introduction.** In this paper we consider the existence of critical points of an even, real valued function on a Banach space. The general approach is by a variant of the Lusternik-Schnirelman theory. An application is made to nonlinear eigenvalue problems of the form

$$(1) \quad \lambda g' = h',$$

where  $g'$  and  $h'$  are gradient operators and where the value of  $\lambda$  is specified. This contrasts with other treatments of (1) by the Lusternik-Schnirelman theory, where (1) is dealt with as an isoperimetric variational problem and consequently the value of  $\lambda$  cannot be specified. Also, it is to be noted that no definiteness assumptions are imposed upon  $g'$ , which contrasts with all other published results concerning (1), except for some results of Amann [13].

Another feature of isoperimetric variational problems is that the parameter appears linearly. It seems likely that certain eigenvalue problems in which the parameter appears nonlinearly could be considered in terms of finding the critical points of a function on a Banach space, although the author has not yet considered any such problems.

We obtain the main theorem of the Lusternik-Schnirelman theory specialized to the case of an even function on a Banach space (Theorem 8). The proof and the preliminary results are largely modifications of results of Palais [2, 5] and Schwartz [1]. However, Theorem 8 is not a specialization of their results. Also, it should be noted that the specialization to a Banach space permits great simplification in the technical details of the proof.

After deriving Theorem 8 we consider corollaries which require special types of nonlinearity in the gradient function, which are related to superlinearity and to asymptotic linearity [12].

Finally, we consider eigenvalue problems of the form (1), with  $\lambda$  specified. Other authors who have, in effect, considered eigenvalue problems with the parameter fixed are Hempel [10], who treats elliptic problems in differential equations, and Coffman [11], who treats Hammerstein integral equations.