

Nerve Axon Equations: II Stability at Rest

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1. Introduction. In a previous paper [3] the behavior of solutions $W(x, t)$ to equations of the form

$$(1) \quad \frac{\partial W}{\partial t} = \begin{bmatrix} \frac{\partial^2 W^0}{\partial x^2} \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} g^0(W) \\ \vdots \\ g^n(W) \end{bmatrix}$$

is studied. Equations of this type arise in neurophysiology in the study of the nerve impulse. The reader is referred to [3] and the references in that paper for background.

Two features required by physiological considerations in a system such as (1) are one, a resting state where $g^0 = \cdots = g^n = 0$ which we take to be at $W = 0$ and two, a nonzero solution to (1) of the form $\phi(x - vt)$ which represents a nerve impulse traveling with velocity v . If $\phi(x - vt)$ is a solution of (1), then $\phi(y)$ satisfies the ordinary differential equation

$$(2) \quad -v \frac{d\phi}{dy} - \begin{bmatrix} \frac{d^2 \phi^0}{dy^2} \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} g^0(\phi) \\ \vdots \\ g^n(\phi) \end{bmatrix}.$$

The linearization of (1) about the resting state $W = 0$ is given by

$$(3) \quad \frac{\partial W}{\partial t} = \begin{bmatrix} \frac{\partial^2 W^0}{\partial x^2} \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} a & r \\ c & B \end{bmatrix} W$$