

# *Scattering Theory for the Acoustic Equation in an Even Number of Space Dimensions*

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**Introduction.** Early in the sixties the authors developed a theory of scattering for the acoustic equation in an odd number of space dimensions, an account of which can be found in [4]. This theory is based on the concept of incoming and outgoing subspaces,  $D_-^\rho$  and  $D_+^\rho$  respectively, where  $D_-^\rho$  [ $D_+^\rho$ ] is defined as the set of initial data for solutions which vanish in the truncated backward [forward] cone:  $|x| < \rho - t, t < 0$  [ $|x| < \rho + t, t > 0$ ]; here  $\rho$  is chosen so that the scattering obstacle is contained in the ball  $\{|x| < \rho\}$ . Denoting the unitary group of solution operators acting on the Hilbert space  $H$  of initial data by  $U(t)$ , the basic properties of  $D_-^\rho$  and  $D_+^\rho$  are:

- 1)  $U(t)D_-^\rho \subset D_-^\rho$  for  $t < 0$ ,  $U(t)D_+^\rho \subset D_+^\rho$  for  $t > 0$ ;
- 2)  $\bigcap U(t)D_\pm^\rho = \{0\}$ ;
- 3)  $\bigcup U(t)D_\pm^\rho = H$ ;
- 4)  $D_-^\rho$  is orthogonal to  $D_+^\rho$ .

From the existence of subspaces  $D_-^\rho$  and  $D_+^\rho$  having properties (1)–(3) it is easy (see Chapters II and V of [4]) to find the corresponding incoming and outgoing translation and spectral representations for  $U$  as well as the explicit form of the scattering matrix. In order to study the singularities of the scattering matrix we introduced the related semi-group of operators (see Chapters III and V of [4]):

$$(1) \quad Z(t) = P_+ U(t) P_- , \quad t \geq 0,$$

acting on the subspace  $K = H \ominus (D_-^\rho \oplus D_+^\rho)$ ; here  $P_-$  and  $P_+$  are the orthogonal projections on the orthogonal complements of  $D_-^\rho$  and  $D_+^\rho$ , respectively. The semi-group property for  $Z$  results from the orthogonality of  $D_-^\rho$  and  $D_+^\rho$ . In our development most of the basic results were obtained by means of the theory of hyperbolic partial differential equations and throughout the Radon transform played a central role (see Chapter IV of [4]).

Somewhat later the corresponding scattering theory problem for the acoustic equation in an even number of space dimensions was treated, first by N. Shenk