

# *Series Expansions with Convergence in the Mean for Dual Laguerre Temperatures*

DEBORAH TEPPER HAIMO

*Communicated by I. N. SNEDDON*

**§1. Introduction.** In a recent paper [5], the author, with F. M. Cholewinski, established that a necessary and sufficient condition that a solution  $u(x, t)$  of the Laguerre differential heat equation

$$xu_{xx} + (\alpha + 1 - x)u_x = u_t$$

have a series expansion in terms of the Laguerre heat polynomials  $p_{n, \alpha}(x, t)$  where

$$p_{n, \alpha}(x, t) = \sum_{k=0}^n \binom{n}{k} \frac{\Gamma(n + \alpha + 1)}{\Gamma(n - k + \alpha + 1)} (xe^{-t})^{n-k} (1 - e^{-t})^k$$

is that  $u(x, t)$  have the Huygens property. It was shown that an additional integrability condition for  $u(x, t)$  characterized those solutions of the Laguerre differential heat equation which are representable in series of  $w_{n, \alpha}(x, t)$ , the temperature transforms of  $p_{n, \alpha}(x, t)$ . Our present goal is to derive conditions for such series representations with convergence in the mean. The results are parallel in part to those derived for the classical heat equation by P. C. Rosenbloom and D. V. Widder [6] and for the generalized heat equation by the author [3].

**§2. Definitions.** For  $\alpha > -1$ , the Laguerre differential heat equation is given by

$$(2.1) \quad \nabla_x u(x, t) = \frac{\partial}{\partial t} u(x, t)$$

where

$$(2.2) \quad \nabla_x f(x) = xf''(x) + (\alpha + 1 - x)f'(x).$$

We denote by  $H$  the class of all  $C^2$  solutions of (2.1) and call a member of  $H$  a dual Laguerre temperature. The fundamental solution of (2.1) is the function