

Trivializations of Orthogonal Bundles

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1. Introduction. It is known that for any vector bundle E over a compact manifold M , there always exists a complementary bundle E' such that $E \oplus E'$ is isomorphic to a trivial bundle $M \times \mathbf{R}^q$ for some positive integer q . The restriction of this isomorphism to E gives rise to an imbedding of E into $M \times \mathbf{R}^q$. In general, there is no formula for such imbedding. However, if E is an orthogonal vector bundle, then a rather nice formula can be constructed. The imbedding depends on the open covering of M and its associated partition of unity. The imbedding and the exterior derivative on $M \times \mathbf{R}^q$ induce a Riemannian connection with respect to the canonically fibre metric of E . Throughout this paper the partition of unity plays an important role. Also some applications to harmonic sections of E have been made.

2. Imbedding of orthogonal vector bundles. Suppose that M is a separable paracompact manifold of dim n , and suppose that E is an orthogonal vector bundle over M of dim m . Let N denote the set of all natural numbers. Let $\mathfrak{U} = \{U_k\}_{k \in N}$ be a countable locally finite open covering of M such that (i) the intersection of any two members of \mathfrak{U} is connected, (ii) E is trivial over each U_k of \mathfrak{U} . Let $\{P_k^2\}_{k \in N}$ be a partition of unity, and let $\{G_{k1}\}$ be the family of transition functions of E associated to the open covering \mathfrak{U} . On each $U_k \in \mathfrak{U}$, we define a bundle map $\varphi_k : E_k = E|U_k \rightarrow M \times \mathbf{R}^\infty$ as follows: Suppose $U_{k_1}, U_{k_2}, \dots, U_{k_r}$ are the only members of \mathfrak{U} intersecting U_k , and suppose that $k_1 < k_2 < \dots < k_{i-1} < k < k_i < \dots < k_r$. Let Φ_k be a $\infty \times m$ matrix whose transpose Φ'_k is defined by

$$\Phi'_k = (0, \dots, P_{k_1} G'_{k_1 k}, \dots, P_{k_{i-1}} G'_{k_{i-1} k}, \dots, P_k G'_{kk}, \dots, P_{k_i} G'_{k_i k}, \dots, P_{k_r} G'_{k_r k}, 0, 0, \dots).$$

We note that Φ'_k is an $m \times \infty$ matrix whose entries are almost all zero and that $P_{k_1} G_{k_1}$ is in the k_1 -th place, and so on. Suppose $\tau_k : U_k \times \mathbf{R}^m \rightarrow E_k$ is a trivialization. Let $\varphi_k : E_k \rightarrow U_k \times \mathbf{R}^\infty$ be a bundle map such that the linear map $\varphi_k \circ \tau_k : U_k \times \mathbf{R}^m \rightarrow U_k \times \mathbf{R}^\infty$ has the matrix representation Φ_k with respect to the