

The Linearized Equations of Water Waves

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Introduction. Let $x' = (x, y) = (x_1, x_2, \dots, x_{n-1}, y)$ denote a variable point in \mathbf{R}^n . Let G be a domain contained in the half space $\{(x, y); x \in \mathbf{R}^{n-1}, y < 0\}$ whose boundary consists of two parts, Γ and Γ' . Γ is an $(n - 1)$ -dimensional subdomain of the plane $y = 0$ and is conceived of as the free surface of a fluid that fills G . Γ' is a fixed boundary through which no fluid can flow. Let

$$\Delta = \sum_{i=1}^{n-1} \frac{\partial^2}{\partial x_i^2} + \frac{\partial^2}{\partial y^2},$$

and let n denote the outward normal to Γ' . Let subscripts denote differentiation. We seek a function $\mu(x', t)$ defined on $\bar{G} \times [0, \infty]$ satisfying the following equations:

$$(0.1) \quad \Delta \mu = 0 \quad \text{for } x' \in G, \quad t \geq 0,$$

$$(0.2) \quad \mu_n = 0 \quad \text{for } x' \in \Gamma', \quad t \geq 0,$$

$$(0.3) \quad \mu_y + \mu_{tt} = 0 \quad \text{for } x' \in \Gamma, \quad t \geq 0,$$

$$(0.4) \quad \mu = \mu^0, \quad \mu_t = \mu_t^0 \quad \text{for } x' \in \Gamma, \quad t = 0.$$

Here μ^0, μ_t^0 are given functions, *i.e.*, the initial conditions of the problem. We think of t as time, which makes its sole appearance as a parameter in the boundary conditions. We call (0.1–0.4) the linearized equations of water waves.

The purpose of this work is to solve (0.1–0.4). In this connection the first general result seems to be that of Finkelstein (see [8]). He considered (0.1–0.4) for $n = 3$, $\Gamma = \mathbf{R}^2$ and G the half-space with a finite number of compact manifolds removed. He explicitly constructed a Green's function for the half-space and used it to reduce the problem of solving (0.1–0.4) to an integral equation. Whereas Finkelstein's construction extends to the case where G is the strip $\{(x, y); x \in \mathbf{R}^{n-1}, -h_0 < y < 0\}$ with a finite number of compact manifolds removed, it certainly fails when G is any more general.

The next result was that of Friedman and Shinbrot (see [3]). They have shown that in the case of so-called normal domains, (0.1–0.4) have a "strong" solution. This means that (0.1), (0.2) and (0.4) are satisfied classically but (0.3) is satisfied