

# *Penetrations into Shadow Regions and Unique Continuation Properties in Hyperbolic Mixed Problems*

JEFFREY RAUCH & MICHAEL TAYLOR

0. This paper studies the solutions of hyperbolic mixed initial boundary value problems with time independent coefficients. The theory of semigroups of operators is used throughout. In Section 1 mixed problems for second order wave equations are considered from the semigroup point of view and the appropriate extension of the groups to spaces of distributions are obtained. Section 2 contains the main results of the paper. Viewing the solution as a signal or disturbance propagating in time, we prove

1. Every open set exterior to the initial disturbance is reached by the signal at some time  $t \geq 0$  (Theorem 2).
2. Every open set is reached by the signal at some time  $t$ ,  $-\infty < t < \infty$  (Lemma 2).
3. For a suitable generalization of the Klein Gordon equation every open set is reached by the signal at some time  $t \geq 0$  (Theorem 5).

The first of these results asserts that there are no perfect shadows and the second is a sort of unique continuation theorem. In Section 3 extensions to systems and to Maxwell's equations are discussed.

1. Let  $\Omega$  be a connected region in  $\mathbf{R}^n$  with smooth, compact boundary. Let  $A = \sum_{|\alpha| \leq 2} a_\alpha(x) D^\alpha$  be a second order elliptic operator. Assume the coefficients  $a_\alpha$  are smooth, real valued, and bounded on  $\bar{\Omega}$ , and assume there is a constant  $C > 0$  such that  $\sum_{|\alpha| = 2} a_\alpha(x) \xi^\alpha \geq C |\xi|^2$  for all  $x \in \bar{\Omega}$ ,  $\xi \in \mathbf{R}^n$ . Here we take

$$D = \left( \frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_n} \right),$$

the real differential operator.

We suppose the domain  $\mathfrak{D}(A)$  is given by regular elliptic boundary conditions and there exists a strictly negative self adjoint operator  $A_0$  with the same principal part as  $A$ , and  $\mathfrak{D}(A) = \mathfrak{D}(A_0)$ . For example,  $\mathfrak{D}(A)$  could be given by Dirichlet or Neumann boundary conditions.